Multiplying Polynomials

Using the Distributive Property to Multiply a Monomial and a Trinomial

Multiplication of a *monomial* and a *polynomial* is simply an extension of the *distributive property*. Make sure that *every term in the parentheses* is multiplied by the *term in front of the parentheses*.

$$3x(2a + 3b - 4c) =$$
$$6xa + 9xb - 12xc$$

← multiply every term in the parentheses by the term $3x$ in front of the parentheses

Typically, mathematicians like to put things in order. They will rearrange the variables in the answer above so that the variables in each term are *alphabetical*. Therefore, the final answer would be as follows.

$$6xa + 9xb - 12xc =$$
$$6ax + 9bx - 12cx$$

Using the FOIL Method to Multiply Two Binomials

When we multiply two polynomials, we extend the distributive property even further to make sure that every term in the *first* set of parentheses is multiplied by every term in the *next* set of parentheses.

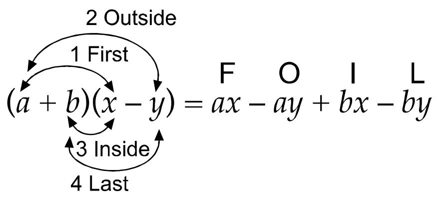
Look carefully at the product below.

$$(a + b)(x - y) = ax - ay + bx - by$$

Notice that both x and y were multiplied by a, and then by b. This is called the **FOIL method** because

- the two **F**irst terms (a and x) are multiplied
- then the two **O**utside terms (a and $-y$) are multiplied
- then the two **I**nside terms (b and x) are multiplied and lastly
- the two **L**ast terms (b and $-y$) are multiplied together.

F **F**irst terms
O **O**utside terms
I **I**nside terms
L **L**ast terms



It is important to be *orderly* when you multiply to ensure that you don't leave out a step. Also, be very careful to watch the positive (+) and negative (–) signs as you work.

Special patterns often occur. Knowing these may help you.

F + O + I + L

$$(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$$

F + O + I + L

$$(a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2$$

F + O + I + L

$$(a - b)(a + b) = a^2 - b^2$$

Alert!

$(a + b)^2 \neq a^2 + b^2$ ← To write this expression in simplest form, the power of 2 is not simply distributed over *a* + *b*. Instead…

$(a + b)^2 = (a + b)(a + b)$ ← $(a + b)^2$ is multiplied by itself, $(a + b)(a + b)$.

Using the Distributive Property to Multiply Any Two Polynomials

Let's look at using the distributive property to do the following.

- multiply a binomial and a trinomial in horizontal form
- multiply two trinomials in horizontal form
- multiply polynomials in vertical form

Example 1

Find the product of a binomial and a trinomial in **horizontal form**.

$$(2a + 5)(3a^2 - 8a + 7) =$$

$$(2a + 5)(3a^2 - 8a + 7) =$$

distributive property

$$2a(3a^2 - 8a + 7) + 5(3a^2 - 8a + 7) =$$

$$(6a^3 - 16a^2 + 14a) + (15a^2 - 40a + 35) =$$

combine

$$6a^3 - 16a^2 + 14a + 15a^2 - 40a + 35 =$$

combine like terms

combine

$$6a^3 - a^2 - 26a + 35$$

Example 2

Find the product of two trinomials in **horizontal form**.

$$(b^2 + 4b - 5)(3b^2 - 7b + 2) =$$

$$(b^2 + 4b - 5)(3b^2 - 7b + 2) =$$

$$b^2(3b^2 - 7b + 2) + 4b(3b^2 - 7b + 2) - 5(3b^2 - 7b + 2) =$$

$$(3b^4 - 7b^3 + 2b^2) + (12b^3 - 28b^2 + 8b) - (15b^2 - 35b + 10) =$$

distributive property

$$3b^4 - 7b^3 + 2b^2 \ + \ 12b^3 - 28b^2 + 8b \ - \ 15b^2 + 35b - 10 =$$

$$3b^4 + 5b^3 - 41b^2 + 43b - 10$$

combine like terms

Example 3

Find the product of polynomials in **vertical form**.

$$(c^3 - 8c^2 + 9)(3c + 4) =$$

Note: There is no c term in $c^3 - 8c^2 + 9$, so $0c$ is used as a placeholder.

$$\begin{array}{r}
c^3 - \ 8c^2 + \ 0c + \ 9 \\
(\text{x}) \qquad\qquad \underline{3c + \ 4} \\
4c^3 - 32c^2 + \ 0c + 36 \\
\underline{3c^4 - 24c^3 + \ 0c^2 + 27c \qquad} \\
3c^4 - 20c^3 - 32c^2 + 27c + 36
\end{array}$$